Fluid horizontal velocity distribution:

$$v = \frac{-Ra^{1/4}}{\sqrt{2}} x^{-2} \left[-4F - 8 \frac{Ra^{1/4}}{\sqrt{2}} yF'x^{-2} \right]$$

$$= -\frac{\eta}{y} [-4F - 8\eta F']. \quad (24)$$

Fin local Nusselt number:

$$Nu = \frac{Ra^{1/4}}{\sqrt{2}} [-\theta'(0)] x^{-2} = \frac{\eta}{y} [-\theta'(0)]. \quad (25)$$

Fin local heat flux:

$$q = \frac{Ra^{1/4}}{\sqrt{2}} [-\theta'(0)] x^{-9} = \frac{\eta \phi_F}{y} [-\theta'(0)]. \quad (26)$$

Fin total heat transfer:

$$q_T = \frac{14 k_F t}{k X_b}. \quad (27)$$

In the present solution it should be noted that the fluid approaches the fin base at $x = 1$ rather than moving in the opposite direction, away from a leading edge at $x = 0$, as it does in most other similarity solutions. The results are valid for long fins, and give a first approximation for heat transfer and fluid flow for finite length fins providing the tip temperature is nearly equal to the bulk fluid temperature and the heat transfer near the tip is an insignificant fraction of the total fin heat transfer. No direct comparison can be made between the present solution and the isothermal vertical flat plate similarity solution because of the lack of a leading edge and the breakdown of the fin conduction equation under isothermal fin conditions.

Acknowledgements—This study was supported by the U.S. National Science Foundation through Grant CME-8003498 and by the Korea Science Foundation. The authors would also like to thank the Engineering Research Institute at Iowa State University for support in the preparation of this article.

REFERENCES

1. E. Schmidt and W. Beckmann, Das Temperatur- und Geschwindigkeitsfeld vor einer Wärme abgebenden senkrechten Platte bei natürlicher Konvektion, *Tech. Mech. Thermo-Dynam. Berl.* **1**, 391–406 (1930).
2. E. M. Sparrow and J. L. Gregg, Similar solutions for free convection from a nonisothermal vertical flat plate, *Trans. Am. Soc. Mech. Engrs* **80**, 379–386 (1958).
3. G. S. H. Lock and J. C. Gunn, Laminar free convection from a downward-projecting fin, *Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **90**, 63–70 (1968).
4. K. T. Yang, Possible similarity solutions for laminar free convection on vertical plates and cylinders, *J. Appl. Mech.* **38**, *Trans. Am. Soc. Mech. Engrs* **82**, 230–236 (1960).
5. P. J. Schneider, Conduction, in *Handbook of Heat Transfer* (edited by W. M. Rohsenow and J. P. Hartnett), pp. (3)112–(3)115. McGraw-Hill, New York (1973).
6. D. Q. Kern and A. D. Kraus, *Extended Surface Heat Transfer*. McGraw-Hill, New York (1972).
7. E. M. Sparrow and S. Acharya, A natural convection fin with a solution-determined nonmonotonically varying heat transfer coefficient, *Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **103**, 218–225 (1981).
8. S. W. Churchill, P. Chao and H. Ozoe, Extrapolation of finite-difference calculations of laminar natural convection in enclosures to zero grid size, *Numerical Heat Transfer* **4**, 39–51 (1981).

Int. J. Heat Mass Transfer. Vol. 26, No. 11, pp. 1721–1726, 1983
Printed in Great Britain
0017-9310/83 $3.00 + 0.00
Pergamon Press Ltd.

AN APPROXIMATE SOLUTION PROCEDURE FOR LAMINAR FREE AND FORCED CONVECTION HEAT TRANSFER PROBLEMS

AKIRA NAKAYAMA and HITOSHI KOYAMA
Department of Mechanical Engineering, Shizuoka University, Johoku, Hamamatsu 432, Japan

and

SEI-ICHI OHSAWA
Tokyo Sanyo Electric Corporation, Oh-izumi, Gunma, Japan

(*Received* 25 *January* 1983)

NOMENCLATURE

$A, \ldots, H$	boundary layer shape factors
C_{fx}	local friction coefficient
f	velocity profile
g_x	streamwise component of gravity
Grx	local Grashof number
I, I_t	functions associated with the deviation from unity
m	parameter for the external velocity variation
m_t	parameter for the ambient temperature variation
n	parameter for the wall-ambient temperature difference variation
Nux	local Nusselt number
Pr	Prandtl number
Rex	local Reynolds number
T	temperature
ΔT_w	wall-ambient temperature difference
u	velocity in x direction
x, y	boundary layer coordinates

Greek symbols

β	coefficient of thermal expansion
δ, δ_t	viscous and thermal boundary layer thicknesses
ζ	boundary layer thickness ratio
η, η_t	similarity variables in y direction
θ	temperature profile

Λ, Λ_f	shape factors associated with the second derivative of the velocity profile
ν	kinematic viscosity
ϕ	wedge half-angle.

Subscripts

c	characteristic scale
e	boundary layer edge
t	thermal boundary layer
w	wall

INTRODUCTION

THE PRESENT note proposes an approximate solution procedure for the prediction of the laminar convective heat transfer through thermal boundary layers. This procedure based on an integral approach is quite general and applicable to both free and forced convection heat transfer problems. The origin of the method stems from the previous work on the free convection flows under non-uniform gravity [1], which subsequently has been extended to the laminar film condensation problem [2]. In the previous free convection study [1], the ratio of the viscous boundary layer thickness to the thermal boundary layer thickness has been treated as an additional unknown to account for the Prandtl number effects. Moreover, an effort has been made to satisfy the conditions on the second derivatives of the velocity and temperature profiles at the wall, which are implicit in the differential form of the momentum and energy conservation equations. These wall conditions (which are not satisfied in usual integral methods in free convection flows) obviously are not trivial since it is the local temperature gradient at the wall which determines the heat transfer rate through the boundary layers.

It will be shown that the same principles can be adopted for the forced convective heat transfer problems. In order to reveal full features of this general solution procedure, the present study first deals with the forced convection over a non-isothermal wedge, and then, the free convection over a non-isothermal flat plate immersed in a thermally stratified fluid, as depicted in Fig. 1. The two problems, although representing quite different physical situations, have common features in their mathematical treatments. Both problems admit similar solutions in the cases which can be specified by two configuration parameters associated with the external and wall conditions.

General expressions for the local Nusselt number are obtained in a closed form for arbitrary values of Prandtl number and the configuration parameters. In fact, the agreement between the present approximate solutions and the available exact solutions turns out to be excellent over a wide range of Prandtl numbers.

FORCED CONVECTION OVER A WEDGE

Without restricting the physical model to the wedge flow of the present concern in Fig. 1(a), the integral relations for the conservation equations may be obtained from a usual control volume analysis upon carrying out the integration toward the outermost boundary layer edge, namely, $0 \leqslant y \leqslant \max(\delta, \delta_t)$, where δ and δ_t are the viscous and thermal boundary layer thicknesses, respectively. After some manipulation, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{\delta}(u_e u - u^2)\,\mathrm{d}y + \frac{\mathrm{d}u_e}{\mathrm{d}x}\int_0^{\delta}(u_e - u)\,\mathrm{d}y = \nu\left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{1a}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{\delta_t} u(T - T_e)\,\mathrm{d}y = -\frac{\nu}{Pr}\frac{\partial}{\partial y}(T - T_e)|_{y=0}. \tag{1b}$$

The boundary layer coordinates (x, y) are aligned along the wall and its normal in a usual manner. u is the velocity in the x direction while the temperature, Prandtl number and kinematic viscosity are denoted by T, Pr and ν, respectively.

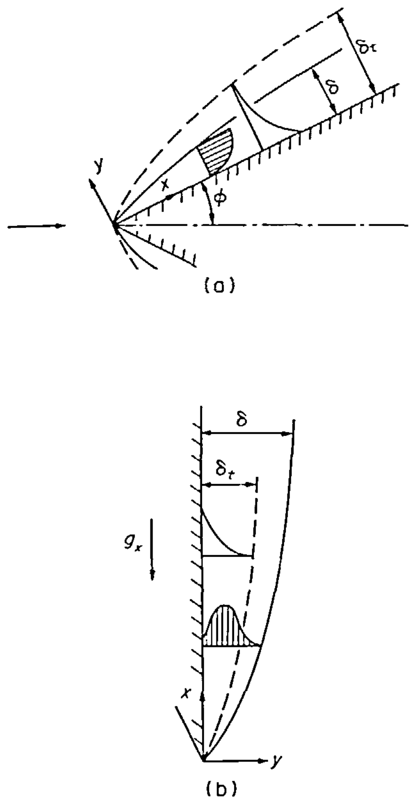

FIG. 1. Physical model and coordinates (a) forced convection over a wedge and (b) free convection over a vertical flat plate.

The subscript e refers to the corresponding boundary layer edge, $y = \delta$ or δ_t, thus, u_e for the free stream velocity and T_e for the ambient temperature which is assumed to be constant in the case of forced convection flows.

As already emphasized, special attention is paid on the conditions along the wall, which virtually govern the local heat transfer rate. The observation on the original differential forms of the conservation equations reveals that the following auxiliary relations must hold at the wall where the convective terms vanish:

$$\nu\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = -u_e\frac{\mathrm{d}u_e}{\mathrm{d}x}, \tag{2a}$$

and

$$\frac{\partial^2}{\partial y^2}(T - T_e)|_{y=0} = 0. \tag{2b}$$

Upon noting that the velocity field in the forced flow is free from the temperature field, equation (1a) may be integrated to obtain the following expression:

$$(\delta/x)^2\, Rex = (2C/G)I, \tag{3a}$$

where

$$Rex = u_e x/\nu, \tag{3b}$$

$$C = \frac{\delta}{u_e}\left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{3c}$$

$$G = \int_0^{\delta}(u_e u - u^2)\,\mathrm{d}y/u_e^2\delta, \tag{3d}$$

$$H = \int_0^{\delta}(u_e - u)\,\mathrm{d}y/Gu_e\delta, \tag{3e}$$

and

$$I = \frac{\int_0^x CGu_e^{3+2H} \exp\left[-\int_0^x (dH/dx)\ln u_e^2\,dx\right]dx}{CGu_e^{3+2H}\exp\left[-\int_0^x (dH/dx)\ln u_e^2\,dx\right]x}. \quad (3f)$$

The shape factor G may be identified as the ratio of the momentum thickness to δ while H as the ratio of the displacement thickness to the momentum thickness. The function I obviously becomes unity in the case of a flat plate at zero incidence. Equation (2a) for the wall condition may now be rewritten as

$$(\delta/x)^2\, Rex = 6\Lambda \Big/ \left(\frac{d\ln u_e}{d\ln x}\right), \quad (4a)$$

where

$$\Lambda = -\frac{1}{6}\frac{\delta^2}{u_e}\frac{\partial^2 u}{\partial y^2}\bigg|_{y=0}. \quad (4b)$$

For the one-parameter family of velocity profiles, the Pohlhausen's polynomial of the fourth degree may be chosen:

$$u/u_e \equiv f(\eta;\Lambda) = (2+\Lambda)\eta - 3\Lambda\eta^2 - (2-3\Lambda)\eta^3 + (1-\Lambda)\eta^4, \quad (5a)$$

where

$$\eta = y/\delta. \quad (5b)$$

In addition to the wall condition given by equation (4b), the function satisfies $f = 0$ at $\eta = 0$ and $\partial f/\partial\eta = \partial^2 f/\partial\eta^2 = 0$, $f = 1$ at $\eta = 1$.

Upon performing integrations on equation (5a), all the shape factors such as C, G and H may be specified in terms of the algebraic functions of Λ alone. Substituting these relations into the RHS of equation (3a), and equating equation (3a) with equation (4a), one obtains an implicit equation for $\Lambda(x)$ which, in general, must be solved by an iterative procedure for a given $u_e(x)$ distribution. However, for the self-similar boundary layers of the present concern, equation (3f) can be reduced to

$$I = \frac{1}{1+(3+2H)m}, \quad (6a)$$

where

$$u_e \propto x^m, \quad (6b)$$

and

$$m = \phi/(\pi - \phi). \quad (6c)$$

ϕ in equation (6c) denotes the wedge half-angle as indicated in Fig. 1(a). Naturally, the above-mentioned implicit equation for Λ reduces itself to a remarkably simple algebraic equation as follows:

$$m = \frac{\Lambda(148 - 8\Lambda - 5\Lambda^2)}{15(56 - 52\Lambda + 10\Lambda^2 + \Lambda^3)}. \quad (7)$$

Thus, the differential equation has been reduced to the simple algebraic equation. It is interesting to note that equation (7) predicts the flow separation ($\Lambda = -2$) at $m = -0.1$. This value is in good accord with -0.091 obtained from the exact solution by Hartree [3]. With the aid of equation (4a), the friction coefficient

$$C_{fx} = \frac{2\nu}{u_e^2}\frac{\partial u}{\partial y}\bigg|_{y=0}$$

may be formulated as

$$C_{fx}\, Rex^{1/2} = 2(2+\Lambda)\left(\frac{m}{6\Lambda}\right)^{1/2}. \quad (8)$$

The variation of the friction coefficient with respect to the configuration parameter m is indicated along with the exact solution by Hartree [3] in Fig. 2. The agreement appears to be excellent.

Having established the velocity field, the energy equation (1b) may now be integrated to yield

$$(\delta_t/x)^2\, Rex = (2E/Pr\, D)I_t, \quad (9a)$$

where

$$E = -\frac{\delta_t}{\Delta T_w}\frac{\partial}{\partial y}(T - T_e)|_{y=0}, \quad (9b)$$

$$\Delta T_w = T_w - T_e, \quad (9c)$$

$$D = \int_0^{\delta_t} u(T - T_e)\,dy/u_e\Delta T_w\delta_t, \quad (9d)$$

and

$$I_t = \frac{\int_0^x D\,\Delta T_w^2 u_e\,dx}{D\,\Delta T_w^2 u_e x}. \quad (9e)$$

ΔT_w denotes the temperature difference between the wall and the ambient fluid. Equation (2b) for the wall condition prompts one to assume

$$(T - T_e)/\Delta T_w \equiv \theta(\eta_t) = 1 - f(\eta_t;0) = 1 - 2\eta_t + 2\eta_t^3 - \eta_t^4, \quad (10a)$$

where

$$\eta_t = y/\delta_t. \quad (10b)$$

Equation (10a) yields $E = 2$ and

$$D = \int_0^1 \theta(\eta_t) f[\min(\eta_t/\zeta, 1);\Lambda]\,d\eta_t$$

$$= \begin{cases} [168(2+\Lambda)\zeta^3 - 180\Lambda\zeta^2 - 27(2-3\Lambda)\zeta + 14(1-\Lambda)]/2520\zeta^4 & \text{for } \zeta \geq 1, \quad (11a) \\ [756 - 126(6-\Lambda)\zeta + 84(4-\Lambda)\zeta^2 - 18(3-\Lambda)\zeta^4 + (14-5\Lambda)\zeta^5]/2520 & \text{for } \zeta \leq 1, \quad (11b) \end{cases}$$

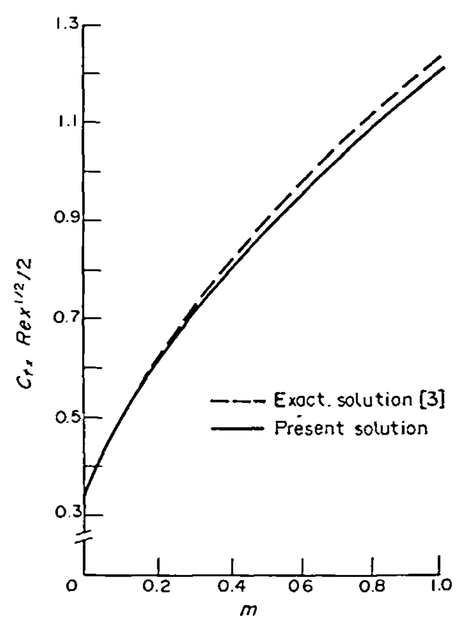

FIG. 2. Friction coefficient in wedge flow.

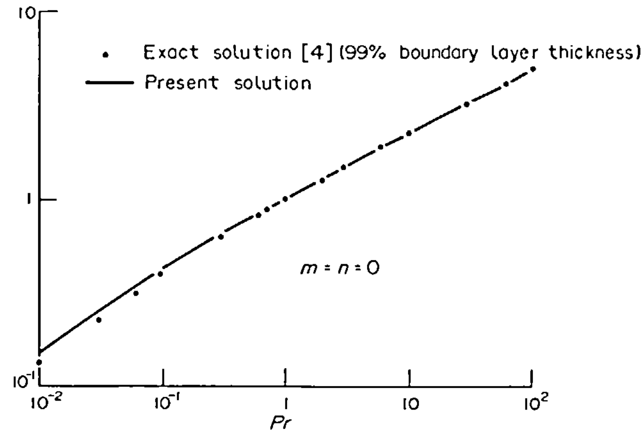

FIG. 3. Boundary layer thickness ratio in wedge flow.

where the boundary layer thickness ratio ζ is introduced as

$$\zeta = \delta/\delta_t. \tag{12}$$

In the case of the similar thermal boundary layers, equation (9c) can be reduced to

$$I_t = \frac{1}{1+m+2n}, \tag{13a}$$

where

$$\Delta T_w \propto x^n. \tag{13b}$$

The function I_t becomes unity in the case of the isothermal flat plate at zero incidence.

Equations (4a) and (9a) may be combined to give

$$Pr = \frac{2}{3}\frac{m}{\Lambda(1+m+2n)}\left(\frac{\zeta^2}{D}\right). \tag{14}$$

Thus, the differential equations for the momentum and energy conservations have eventually been reduced to a pair of simple algebraic equations (7) and (14) in the case of the similar boundary layers. For given m, the shape factor Λ can be obtained from equation (7), and all the coefficients in $D(\zeta;\Lambda)$ may be evaluated according to equations (11). Once this is done, equation (14) may be used to obtain ζ for a given Pr and the configuration parameters m and n. This essentially completes the solution of the problem.

With the aid of equation (4a), the following expression may be derived for the local Nusselt number $Nux = Ex/\delta_t = 2\zeta x/\delta$ which is of primary interest in the present study:

$$Nux/Rex^{1/2} = 2\zeta\left(\frac{m}{6\Lambda}\right)^{1/2}. \tag{15}$$

The boundary layer thickness ratio ζ and the proportional constant associated with Nux are plotted in Figs. 3 and 4 for the case of the isothermal wall, namely, $n = 0$. The numerical values from the exact solutions [4, 5] are also shown in Figs. 3 and 4 for comparison. The agreement of the present approximate solutions with the exact solutions is seen to be extremely good.

FREE CONVECTION OVER A VERTICAL FLAT PLATE IMMERSED IN A THERMALLY STRATIFIED FLUID

The same principles can be adopted for the free convection problem. As performed in the forced convection problem, the integral relations for the conservation equations have been obtained from a control volume consideration within the thickness from the wall to the outermost boundary layer edge. Upon noting $u = 0$ for $y \geqslant \delta$ and $T = T_e$ for $y \geqslant \delta_t$ (thus, saving some part of the integration), one obtains the following momentum and energy equations:

$$\frac{d}{dx}\int_0^{\delta} u^2\,dy = \beta g_x \int_0^{\delta_t}(T-T_e)\,dy - v\left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{16a}$$

$$\frac{d}{dx}\int_0^{\min(\delta,\delta_t)} u(T-T_e)\,dy + \frac{dT_e}{dx}\int_0^{\delta} u\,dy = -\frac{v}{Pr}\frac{\partial}{\partial y}(T-T_e)|_{y=0}. \tag{16b}$$

In equation (16a), the thermal expansion coefficient is given by β while the streamwise component of the gravitational acceleration is denoted by g_x which, of course, is constant for the flat plate of the present concern, but, may, in general, be a function of x. For the constant ambient temperature, equations (16) become identical to those obtained in the previous study [1].

The auxiliary relations for the required wall conditions are given upon eliminating the convection terms from the original differential equations:

$$v\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = -\beta g_x \Delta T_w, \tag{17a}$$

and

$$\frac{\partial^2}{\partial y^2}(T-T_e)|_{y=0} = 0. \tag{17b}$$

The functions for the velocity and temperature profiles are

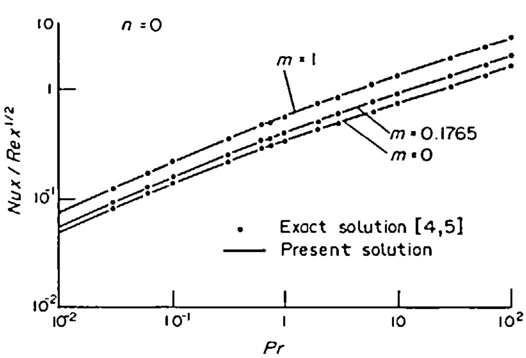

FIG. 4. Nusselt number in wedge flow.

introduced as

$$u/u_c = f(\eta), \tag{18a}$$

and

$$(T-T_e)/\Delta T_w = \theta(\eta_t), \tag{18b}$$

where the characteristic velocity u_c and the wall and ambient temperature difference ΔT_w are functions of x alone. Equation (17a) may now be written alternatively as

$$u_c = \beta g_x \Delta T_w \delta^2/\nu\Lambda_f, \tag{19a}$$

where

$$\Lambda_f = -\left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0} \tag{19b}$$

As equations (18) and (19a) are substituted into equations (16), equations (16a) and (16b) may be solved for two unknowns, the viscous boundary layer thickness δ and the boundary layer thickness ratio ζ. For this purpose, one may regard both equations (16a) and (16b) as the differential equations for δ^4 and obtain two distinct closed form expressions for δ^4. The resulting expressions run as follows:

$$(\delta/x)^4\, Grx = \frac{4\Lambda_f^2}{5A}\left(\frac{B}{\zeta}-\frac{C}{\Lambda_f}\right)I = \frac{4\Lambda_f E}{3Pr}\,\frac{\zeta}{D}\,I_t, \tag{20a}$$

with

$$Grx = \beta g_x \Delta T_w x^3/\nu^2, \tag{20b}$$

$$I = \frac{\int_0^x [(B/\zeta)-(C/\Lambda_f)](g_x\Delta T_w)^{3/5}\,dx}{[(B/\zeta)-(C/\Lambda_f)](g_x\Delta T_w)^{3/5}x}, \tag{20c}$$

and

$$I_t = \frac{\int_0^x \zeta(Dg_x\Delta T_w^5)^{1/3}\exp\left\{(4F/3)\int_0^x [(dT_e/dx)/(D\Delta T_w)]\,dx\right\}dx}{\zeta(Dg_x\Delta T_w^5)^{1/3}\exp\left\{(4F/3)\int_0^x [(dT_e/dx)/(D\Delta T_w)]\,dx\right\}x}, \tag{20d}$$

where

$$A = \int_0^1 f(\eta)^2\,d\eta,\quad B = \int_0^1 \theta(\eta_t)\,d\eta_t,\quad C = \left.\frac{df}{d\eta}\right|_{\eta=0}, \tag{21a,b,c}$$

$$D = \int_0^{\min(1/\zeta,1)} f(\eta)\theta(\zeta\eta)\,d\eta,\quad E = -\left.\frac{d\theta}{d\eta_t}\right|_{\eta_t=0}\quad \text{and}\quad F = \int_0^1 f(\eta)\,d\eta. \tag{21d,e,f}$$

The following third order polynomials are proposed to determine the coefficients A–F:

$$f(\eta) = \eta(1-\eta)^2, \tag{22a}$$

and

$$\theta(\eta_t) = 1-\frac{3}{2}\eta_t+\frac{1}{2}\eta_t^3. \tag{22b}$$

The velocity profile meets the conditions $f = 0$ at $\eta = 0$ and $f = df/d\eta = 0$ at $\eta = 1$ while the temperature profile satisfies not only $\theta = 1$ at $\eta_t = 0$ and $\theta = d\theta/d\eta_t = 0$ at $\eta = 1$, but also the required condition on the second derivative at the wall, namely, equation (17b). The proposed profiles give $\Lambda_f = 4$, $A = 1/105$, $B = 3/8$, $C = 1$, $E = 3/2$, $F = 1/12$ and

$$D = \begin{cases}(42\zeta^2-35\zeta+9)/420\zeta^4 & \text{for } \zeta \geq 1, \quad (23a)\\ (35-21\zeta+2\zeta^3)/420 & \text{for } \zeta \leq 1. \quad (23b)\end{cases}$$

Upon substituting these coefficients into equation (20a), the last two expressions in equation (20a) yield an implicit equation for $\zeta(x)$, which can be solved iteratively for given Pr, $g_x(x)$, $T_e(x)$ and $\Delta T_w(x)$. Such iterative claculations, although for the constant ambient temperature, have been performed in the previous study on the free convection [1].

The self-similar boundary layers of the present concern may be characterized by two constant configuration parameters m_t and n such that

$$T_e - T_c = m_t \Delta T_w, \tag{24a}$$

and

$$\Delta T_w \propto x^n, \tag{24b}$$

where T_c is any constant reference temperature. The parameter m_t together with n characterizes the ambient temperature variation. For example, $m_t = 0$ corresponds to the constant ambient temperature while $m_t = -1$ to the constant wall temperature. Since ΔT_w is assumed to be positive, and $dT_e/dx \geq 0$ for the environment to be stable, only on the combinations of m_t and n satisfying $m_t n \geq 0$ are of physical interest. For such similar boundary layers, the functions I and I_t reduce themselves to

$$I = \frac{1}{1+(3/5)n}, \tag{25a*}$$

and

$$I_t = \frac{1}{1+(1/3)[5+(m_t/3D)]n}. \tag{25b*}$$

The substitution of the above equations into the last two expressions in equation (20a) leads to the following algebraic equation for ζ:

$$Pr = \frac{1+(3/5)n}{1+(1/3)[5+(m_t/3D)]n}\left\{\frac{\zeta}{21D[(3/\zeta)-2]}\right\}. \tag{26}$$

For given Pr and the configuration parameters m_t and n, one can readily obtain ζ from equation (26) with $D(\zeta)$ given by equations (23). Equation (26) indicates that ζ varies from 0 to 3/2 as Pr goes from 0 to ∞. The variation of ζ for the constant ambient temperature can be found in the previous report [1].

Once ζ is determined in this fashion, the local Nusselt number $Nux = Ex/\delta_t = (3/2)\zeta x/\delta$ may be calculated from

$$Nux/Grx^{1/4} = \frac{3}{2}\zeta\left\{\frac{1+(3/5)n}{168[(3/\zeta)-2]}\right\}^{1/4}. \tag{27}$$

The variations of the local Nusselt number are indicated in Figs. 5 and 6 for the constant wall temperature at $m_t = -1$ and the increasing wall temperature at $m_t = 1$, respectively. Pictorial representation of the wall and ambient temperature distributions are given on the lower right of the figures. The

* I and I_t defined in equations (20c) and (20d) are somewhat different from those in ref. [1].

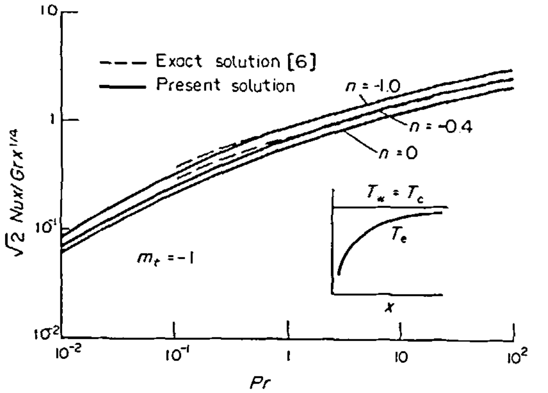

FIG. 5. Nusselt number in free convection over an isothermal plate.

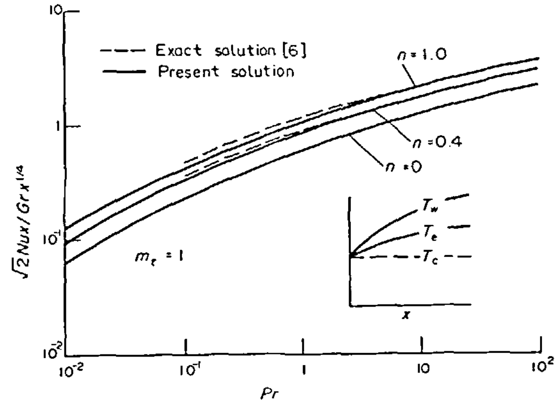

FIG. 6. Nusselt number in free convection over a nonisothermal plate.

exact solutions ($0.1 \leqslant Pr \leqslant 20$) obtained by Yang *et al.* [6] are also plotted for the purpose of comparison. The curves in Figs. 5 and 6 include the case of $n = 0$ for the constant wall and ambient temperatures as a reference. It is seen that the present solution for $n = 0$ almost coincides with the exact solution. Although the curves for non-zero n seem to deviate from the exact solutions as Pr becomes small, an excellent agreement has been maintained for the high Prandtl number cases.

CONCLUDING REMARKS

In the case of the similar boundary layers, the present solution procedure (applicable to both free and forced convection flows) reduces the conservation equations to a simple algebraic equation among the boundary layer thickness ratio, Prandtl number and the configuration parameters. Thus, an algebraic calculation of the boundary layer thickness ratio for given Prandtl number and the configuration parameters essentially completes the solution of the problem. It is believed that the excellent agreement of the present approximate solutions with the exact solutions is primarily due to the practice employed to meet the conditions on the curvatures of the velocity and temperature profiles at the wall.

The mathematical simplification achieved in the present study can be exploited for the speedy and accurate estimation of the local heat transfer rate through the thermal boundary layers.

Acknowledgement—The authors would like to express their sincere thanks to Mr E. Makita for a number of invaluable suggestions on this report.

REFERENCES

1. H. Koyama and A. Nakayama, An integral method in free convection flows under non-uniform gravity, *Heat Mass Transfer* 9(3), 151–158 (1982).
2. A. Nakayama, H. Koyama and S. Ohsawa, An integral method in laminar film condensation on plane and axisymmetric bodies, *Heat Mass Transfer* 9(6), 443–453 (1982).
3. D. R. Hartree, On an equation occurring in Falkner and Skan's approximate treatment of the equations of the boundary layer, *Proc. Camb. Phil. Soc.* 33, 223–239 (1937).
4. F. M. White, *Viscous Fluid Flow*, pp. 273, 283. McGraw-Hill, New York (1974).
5. H. L. Evans, Mass transfer through laminar boundary layers—similar solutions of the b-equation when $B = 0$ and $\sigma \geqslant 0.5$, *Int. J. Heat Mass Transfer* 3, 26–41 (1961).
6. K. T. Yang, J. L. Novotny and Y. S. Cheng, Laminar free convection from a nonisothermal plate immersed in a temperature stratified medium, *Int. J. Heat Mass Transfer* **15**, 1097–1108 (1972).